Ground clutter processing for airborne radar in a Compressed Sensing context

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***Abstract*—Changes in the context of airborne radar processing implies more and more improvements in terms of radar time management. New principles like Compressed Sensing are able especially to take into account modern situations of multiple echoes, provided some dedicated signal is sent by the radar. However, the presence of ground clutter in the signal received by the radar affects deeply the efficiency of the reconstruction treatment. Then the principle retained here has to be extended. Moreover, fine range and Doppler resolutions are required for identification. Thus we define and build a more accurate model of ground clutter. We show that our original model satisfies necessary hypothesis for known reconstruction and clutter separation procedure.**

I. Introduction

The process conventionally used to estimate parameters of targets detected by the radar is matched filtering, it is optimal for the detection of a single point target [1], [2]. Compressed sensing would define a new treatment for the radar, also performing in the multi-target case.

When the expected metric resolution is accurate, each reflecting element in the propagation field of the wave is modeled as a set of multiple point targets, each of them generating an echo of the wave emitted by the radar, modulated in time by a delay proportional to its distance from the radar, in frequency by the Doppler effect proportional to its relative velocity, and in amplitude by a factor depending on its local electromagnetic signature. A multiple target scene can thus be represented in a 2D discrete grid, indexed by testable Doppler frequencies and time delays, in which the amplitude of each localized point target is spotted. Apart from these points, any other coordinate of the grid is affected to a zero value, meaning that no reflecting point has been detected at the corresponding range and velocity. Since the grid is generally large, the number of reflecting elements appears to be very weak in comparison to the size of the research space and the grid is likely represented by a sparse matrix. That's why Compressed Sensing seems to be adapted to such situation.

But this hypothesis is valid when only few airborne objects, like planes, are present in the field of the radar emission. However, a large surface of the ground beneath the airborne platform often intercepts the electromagnetic wave. This defines a geometrical and physical intersection in which many point elements reflect with non negligible amplitudes the signal back to the radar. Therefore the exact representation of a global reflecting scene included both planes and ground clutter would be composed of many nonzero point in the Doppler-delay discrete grid, so that the parsimony of the scenario in the research space is no more valid. From this perspective, one wish to extend the mathematical context of Compressed Sensing to a procedure relevant to the separation of the ground clutter and the so-called ”useful signal” referring to airborne targets. This can be tractable provided the sub-matrix representing the clutter within the grid supports algebraic conditions consistent with some procedure for reconstruction and separation. The approach retained here is the ”sparse plus low rank procedure”, that consists in modeling the unknown features of the reflecting scene into the sum of a sparse matrix and a low rank matrix representing respectively the airborne objects and the ground in the radar propagation field. One main objective of this paper is to check if one can build a model for the reflecting ground to make it representable by a low rank matrix.

Current geometrical and physical modeling for wave echoes from the ground is not accurate enough, because of geometrical approximations to simplify the modeling. For example, iso-velocity hyperbola on the ground are often approximated by their asymptotes. More exact modeling is however required to assess the algebraic properties of the underlying range-Doppler matrix. For this purpose, the complete writing of the corresponding sparse plus low rank procedure for the reconstruction and separation of airborne and ground echoes is being set up. Then the step by step construction of the geometrical and physical model for the ground clutter is build. Finally, we compare the representation of the ground with the ones already obtained for current model and test its mathematical properties as a matrix.

II. Application of sparse plus low rank approach to the radar reconstruction and separation

A. The basic Compressed Sensing approach for the radar

Suppose we only consider the presence of airborne targets and not the ground effects, then the procedure to detect, locate and indentify the point objects in the propagation field of the radar signal using the Compressed Sensing principle [3] writes :

$$y = \Phi\alpha + w \qquad (1)$$

where :

- $y \in \mathbb{C}^m$ is the vector of measured data, obtained from the radar received signal.
- $\Phi \in \mathbb{C}^{m \times P}$ is the matrix of measures, entirely determined by the definition parameters of the electromagnetic signal emitted by the radar.
- $\alpha \in \mathbb{C}^P$ is the unknown representation of the observed target scene in the reconstruction grid, recast into a large vector.
- $w \in \mathbb{C}^m$ is the vector noise samples, supposed to be a centered multi-normal vector with known covariance matrix.

The number of measurements m and the number of features P (product of the numbers of testable Doppler frequencies and time delays in the reconstruction grid) are integers such that $m \ll P$. Moreover, the hypothesis of parsimony for α writes $|supp(\alpha)| \leq s$, where $supp(\alpha)$ is the set of the nonzero elements indexes in α and $s \in \mathbb{N}$ is a supremal bound for the number of reflecting point targets in the observed scene, that also satisfies the relation $s \ll P$. There are several interests in this approach. First, multiple targets cases are directly taken into account in the problem formulation, conversely to the matched filtering procedure. Then the linear system to be solved is under-determined, reducing the number of stored measurements and the flow rate.

To solve 1 under the parsimony assumption, dedicated programs such Basis Pursuit and LASSO can be used. The latter consists in an iterative algorithm solving the following convex non-differentiable problem, with the previously defined parameters of the radar context :

$$\underset{x \in \mathbb{C}^P}{min} \frac{1}{2} \left|\left|y - \Phi x\right|\right|_2^2 + \lambda \left|\left|x\right|\right|_1 \quad (2)$$

λ is a cross-validation determined coefficient, so that the computed solution corresponds to the sparse scene consistent with the noisy data stored by the radar. The operator $||.||_1$ is the convex relaxation of the function associating to a given vector in $\mathbb{C}^P$ its number of nonzero elements. The latest problem is well-posed provided the matrix of measure Φ satisfies specifical properties. Hopefully, it is possible to define electromagnetic signals for the radar emission that lead to suitable matrices Φ, with satisfying ratio $\frac{s}{m}$. In the current case, matrices Φ are obtained from extended Fourier matrices of size P.

However, 2 is not valid for situations where ground echoes are present in the signal received by the radar, so that the representation of the scene x is not sparse anymore. The main idea to extend 2 for reconstruction and separation in a ground clutter case is to state that the global scene $x = L + S$ is the sum of ground clutter L and the airborne reflecting objects S contributions.

B. *Reconstruction and separation procedure set up*

Consider the decomposition $x = L + S$. The expected procedure supposes both that S is sparse and that L is a matrix of low rank. Thus, if S can be represented as well by a sparse matrix as by a sparse vector, L only supports a matrix form. Since the two data are summed to each other, S has also to be represented by a matrix, by homogeneity. Then the program to be determined admits two matrix variables, contrary to 2 that admits only a vectorial variable. This also implies changes in the expression of the measure operator, previously defined as the matrix Φ.

Just as the program 2 uses the convex relaxation $||.||_1$ to express the sparsity of the sought solution, the sparse plus low rank approach also employ the convex relaxation of the matrix rank function (non convex), i.e. the nuclear norm $||.||_*$, defined as the sum of the singular values modules of the matrix. The sparse plus low lank program we aim to apply to the radar reconstruction and separation writes :

$$\underset{L,S \in \mathbb{C}^{p_1 \times p_2}}{min} \frac{1}{m} \sum_{j=1}^{N} \left(y_j - < \tilde{\Phi}_j, L + S > \right)^2 + \mu \left|\left|L\right|\right|_* + \gamma \left|\left|S\right|\right|_1 \quad (3)$$

where p_1 and p_2 are two integers satisfying $P = p_1 p_2$. For any $j \in [\![1; N]\!]$, $\tilde{\Phi}_j \in \mathbb{C}^{p_1 \times p_2}$ and $<, >$ nominates a scalar product on $\mathbb{C}^{p_1 \times p_2}$. The coefficients μ and γ are obtained by a cross-validation procedure.

It appears that, if p_1 and p_2 refer to the numbers of testable time delays and Doppler frequencies, if, for any $1 \leq j \leq N$, $\tilde{\Phi}_j$ is the recast of the $j-$th row of Φ into a complex $p_1 \times p_2$ matrix and if $<, >$ refers to the Frobenius scalar product on $\mathbb{C}^{p_1 \times p_2}$, then the quadratic term $\sum_{j=1}^{N} \left(y_j - < \tilde{\Phi}_j, L + S > \right)^2$ in 3 corresponds to the L_2-norm of the noise vector $||y - \Phi vec(x)||_2^2$ in 2, where $vec(x)$ refers to the suitable recast of the sum $L + S$ into a vector. The factor $\frac{1}{2}$ affected the quadratic term is replaced by $\frac{1}{m}$ for computational means.

Due to mathematical tools in [4] and [5], the fact that any $\tilde{\Phi}_j, 1 \leq j \leq N$ comes from some Fourier operator is relevant to the existence of convergent iterative procedures solving 3, and therefore computing the researched features in S.

Yet, we still have to establish that the echoes from the ground clutter can be represented by a low rank matrix. That matrix won't have strictly a low rank but it will admit an enough decreasing singular value decomposition, so that 3 may be applied to our radar context.

III. Build of an exact ground clutter model

A. *The phenomenon*

The radar antenna is a circular electronic scanning antenna, that concentrates and directs the wave energy in the wished direction of emission. In fact, many lobes are generated by the antenna during the emission : the main lobe oriented by the fixed direction of emission, and secondary lobes of lower energy, sent in various directions around the main lobe. Each of them can be reflected by the ground beneath the radar. Even if the energy transported by a secondary lobe is significantly smaller than the one transported by the main lobe, any lobe can generate a strong amplitude echo towards the radar, following its relative direction with respect to the ground surface.

Any point on the ground surface intercepted by a radar lobe will reflect the wave with a retro-diffusion amplitude given by the radar equation. In our model, each of these points is spotted by its angular coordinates, the bearing angle θ_g and the angle of site θ_s. We suppose that the ground is straight and horizontal and we consider that the plane remains horizontal during its flight, so that its altitude H remains constant.

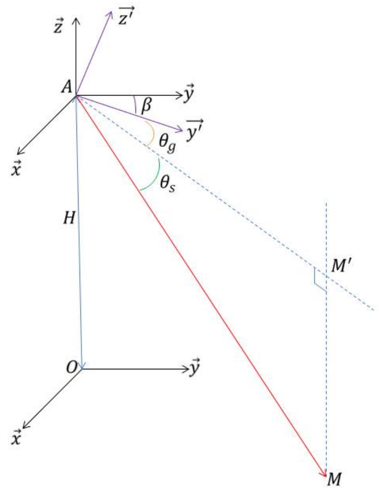

Fig. 1: A reflecting point on the ground represented by its angular coordinates with respect to the center of the radar antenna

On the scheme ahead, the center of the radar antenna is spotted by the point A, its orthogonal projection on the ground is O and the velocity vector of the plane is spotted by the horizontal vector $\vec{y'}$. The point M belongs to the wave illuminated surface on the ground, and $M^{'}$ is its orthogonal projection on the geometric plane $(A, \vec{x}, \vec{AM'})$. M is spotted by its bearing angle $\theta_g = (A, \vec{y'}, \vec{AM'})$ and its angle of site, or elevation, $\theta_s = (A, \vec{AM'}, \vec{AM})$.

The case previously illustrated is very general. The plane direction of flight has an inclination $\beta = (A, \vec{y}, \vec{y'})$ with the horizontal parallel to the ground intercepting A. In what follows, the plane trajectory remains horizontal so that $\beta = 0$ and $\vec{y} = \vec{y'}$.

We now wish to carry out a discretization of the surface of the ground according to the curves representing points of the ground at equal distance from the radar and points of the ground having the same radial speed with respect to the radar.

B. The model

The ground clutter is the sum of the echoes of all the wave-illuminated point on the ground, each of them being characterized by its time delay and Doppler frequency, proportional to its distance and speed from the radar respectively.

1) Determination of iso-distant curves: The set of points of space situated at the same distance D_i of the radar is the surface of the sphere having as center the center of the antenna of the radar and for radius D_i. The points of the ground situated at the distance D_i of the radar are thus located at the intersection of this sphere and the ground.

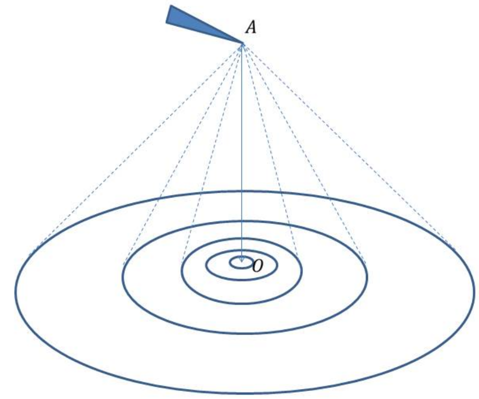

Fig. 2: Iso-distant curves on the ground

The horizontal ground is thus partitioned into several concentric circular crowns associated with the testable times delays indexing the reconstruction grid.

2) Determination of iso-velocity curves: The determination of the points having the same radial velocity V_j with respect to the radar is more complicated. Using the mathematical definition of the radial velocity of one point with respect to another and the vectorial parameterization defined above, calculations show that the points we are looking for are located on the surface of a cone centered on A, which axis is the velocity vector of the aircraft and which angular aperture α depends on the radial velocity under consideration.

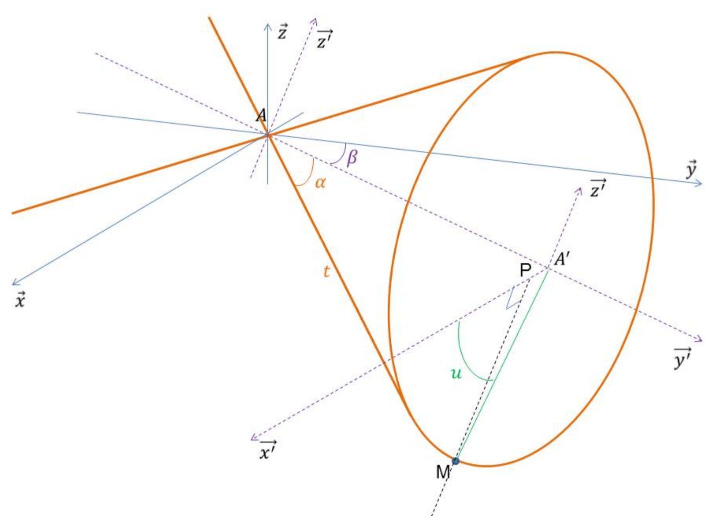

Fig. 3: Iso-velocity curves on the ground

The figure of the intersection of the iso-speed cones with the ground depends on the nature of the flight. When the aircraft remains horizontal, the common axis of all these cones is parallel to the ground. By applying the conic theory on a geometric plane, the intersection of one of these cones with the ground is a hyperbola, originating at the point O. Two distinct hyperbolas never intersect, and the more the associated radial velocity to a hyperbola is high, the more this hyperbola is flared and distant from the origin O.

Until now, the iso-speed curves were only defined in an approximate way in industrial work on the clutter of soil. The exact calculation of their parameters thus constitutes a novelty and allows a more precise modeling of the echoes of soil before being able to test its algebraic properties and take them into account in the simulations realized with dedicated electromagnetic signals.

Combining the results from the studies of iso-distant and iso-speed curves leads to the following discretization of the ground, obtained from numerical computations from *Matlab* :

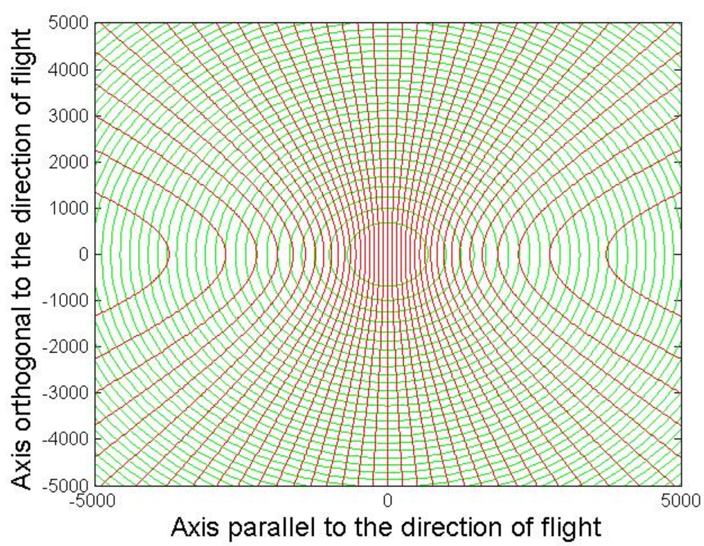

Fig. 4: Discretization of the horizontal ground beneath the radar into Doppler-delay resolution cells

We see in this figure appear different types of geometric surfaces, resulting from the intersection of a crown surface between two successive circles and a surface between two successive hyperboles.

The next step to valid the previous model is to determine with which amplitude each element of that structure reflects the radar wave.

3) Computation of the ground clutter: As the radar balance shows, the amplitudes of ground elements depend on their surface. This is why I have determined the expression of area of each of the infinitesimal surfaces resulting from the above division. This is why we determine the expression of area of each of the infinitesimal surfaces resulting from the above division. Moreover, to satisfy the modeling of the ground clutter as the sum of the echoes of scattering centers, we associate each of these surfaces with their geometric barycenter, which we also calculate analytically. The barycenter of the surfaces are represented by blue stars in the next figure:

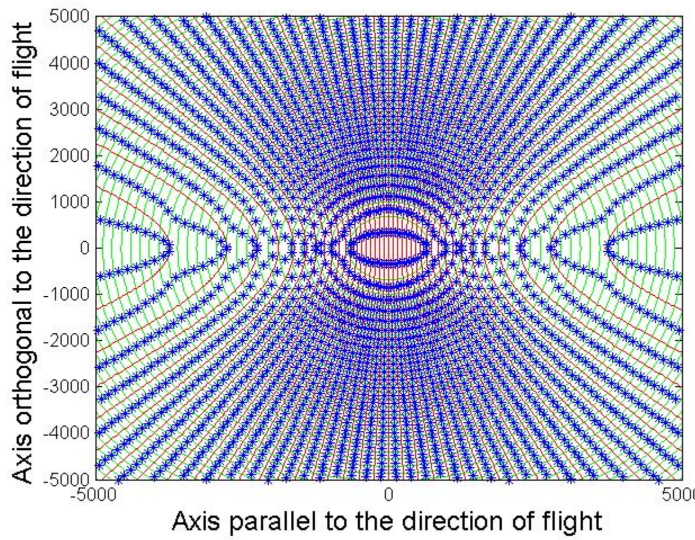

Fig. 5: Repartition of resolution cells barycenter

Using the Cartesian and angular coordinates of these points, we can recalculate all the factors in the expression of the amplitude of these points (distance, antenna gain, reflectivity). We rely on fixed flight parameters for numerical simulations: a platform altitude of 5000*ft*, a carrier speed of 240*m/s*, a viewing angle of the radar given by bearing angle $\theta_g^0 = 30^o$ and angle of site $\theta_s^0 = -1^o$. Fig. 4 and 5 are geometrical modelings of ground clutter, that is sensed by the radar through a waveform that is generally ambiguous in range and velocity. Next figure shows the ambiguous clutter map obtained from Fig. 5 and a periodic waveform with pulse repetition frequency :

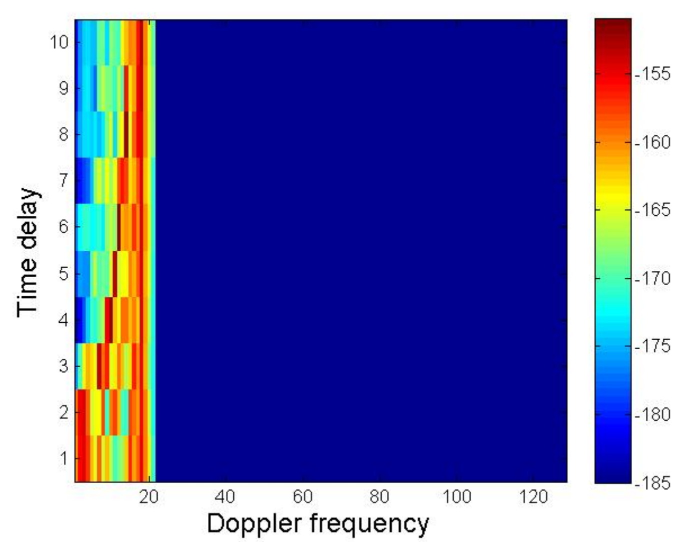

Fig. 6: Ambiguous ground clutter map

The figure we obtained is consistent with real data analyzed previously. We can notice the clear area beyond the Doppler corresponds to the speed of the aircraft, as well as the vertical line corresponds to the Doppler of the elements of the ground illuminated by the main lobe.

The new modeling thus being validated, we can check to test its mathematical properties and to use it in the simulations using more general waveforms such as frequency-hopping waveforms.

IV. Validation of the sparse and low rank approach for the separation of airborne echoes and the ground clutter

We found out that a clear dissociation appears on the map previously obtained between the ground clutter below a certain index and the so-called clear zone beyond this index and in which only airborne targets could appear.

These results justify the sparse plus low rank procedure 3, that can readily be implemented for ground clutter separation from the useful targets signals, while the simpler Compressed Sensing procedure 2 would be used on the clear zone. This is conceivable insofar as the limit between these two zones is defined by a threshold entirely characterized by the flight parameters. Our objective here is to be able to achieve the global reconstruction of the reflective objects and the dissociation of the ground and possible aerial targets in the first zone.

We validated our model of ground clutter by establishing a grid obtained from a classic pulse train radar signal. Such a signal is not adapted to the global Compressed Sensing approach. We suppose now that the radar emits a step frequency waveform, whose carrier frequency varies from one pulse to the next [6], [7]. General advantages induce us to test this family of waveform :

- These waveforms are relatively simple to generate.
- Their form is very general and leaves a large margin of choice in their definition parameters, which can make them adaptable to different types of missions.
- Thanks to the agility of the frequencies allocated to the pulses, these waveforms can have high bandwidths allowing to wait for the desired distance resolutions.

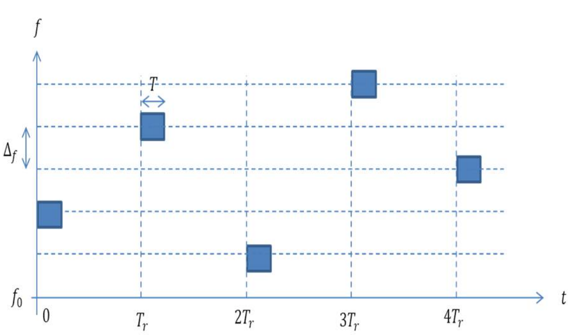

Fig. 7: Time-frequency graph of a step frequency waveform

A radar transmitting such a signal also has the property of being more resistant to jamming and listening. It is in this perspective that we direct our study towards this type of waveforms.

Let us calculate the *SVD* of the matrix representing the area of ground clutter in the search space indexed by the reconstruction grid, obtained from the emission of dedicated step frequency signal.

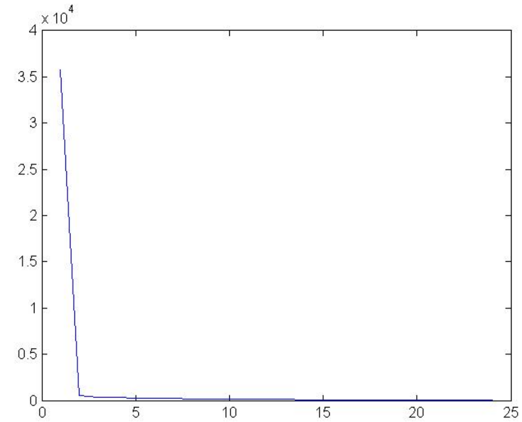

Fig. 8: Singular Value Decomposition of the ground clutter matrix

This figure shows a decrease in the singular values of the matrix in agreement with those of the so-called low rank matrices for which the sparse plus low rank procedure has proved conclusive. This motivates the application of this approach to the detection, location and identification of airborne targets in the presence of clutter in the context of Compressed Sensing radars.

V. Conclusion

We have justified a mathematical approach to handle with the radar procedure of reconstructing and separating from ground clutter the airborne objects that reflected the emitted electromagnetic radar signal. We defined a more accurate model for the ground clutter, in order to make it suitable for the expected resolution of reconstruction and adapted to the sparse plus low rank procedure. Checking the obtained result has able to valid the use of the considered approach with the new model of ground reflection and for emission of a radar signal consistent with Compessed Sensing context. Future work using effective algorithms will be used to rigorously validate this approach for the detection, location and identification of multiple targets in the presence of ground echoes.

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